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KQQKQQ AND THE KASPAROV-WORLD GAME

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USA, Switzerland and the UK

ABSTRACT

The 1999 Kasparov-World game for the first time enabled anyone to join a team playing against a World Chess Champion via the web. It included a surprise in the opening, complex middle-game strategy and a deep ending. As the game headed for its mysterious finale, the World Team requested a KQQKQQ endgame table and was provided with two by the authors. This paper describes their work, compares the methods used, examines the issues raised and summarises the concepts involved for the benefit of future workers in the endgame field. It also notes the contribution of this endgame to chess itself.

1. INTRODUCTION

The contest between Kasparov and the World began on June 21st, 1999 and was essentially a novel correspondence game played at the regulated pace of one ply per day. *The World* was a web-enabled group led by GM Danny King as moderator and four talented, young coaches, together with a bulletin board of FIDE-rated and rateable analysts and chess enthusiasts. Moves were voted for by the participating public along democratic principles (Marko and Haworth, 1999). The game itself was remarkable and lived up to the occasion in all three phases; innovative moves led to a dynamic, unbalanced position requiring precise play by different forces.

As early as move 10, it was clear that the game was likely to go into a complex ending, and after move 39, this could still have been KRBKBN or KQPKQPP. Soon it was down to the royal pieces and their foot soldiers. Defending a notorious QP-ending, the World team called for that utopia of perfect information, an *Endgame Table* (EGT), also referred to as a *Database* (Van den Herik and Herschberg, 1986) and a *Tablebase* (Edwards and the Editorial Board, 1995).

The first request, for KQPKQPP, was quickly seen to be unrealistic but given its prolonged fight for a draw, the World Team had shown great restraint in not asking for EGT help much earlier. The next priority was for the KQPKQP EGT which needed at least the KQQKQQ, alias *4Q*, if not all fifteen KQxKQy EGTs.

Stiller (1992, 1995 and 1996) had created some 41 pawnless 6-man, *White win/no-win* EGTs including all KQxKQy endgames except those with *xy* as *QB*, *QN*, *RQ*, *BQ* and *NQ*. Unfortunately, none survived due to a lack of file space and they were sorely missed by the World Team. Nevertheless, Stiller's remaining summary information usefully underlined the feasibility of computing 4Q with its maximal depth to a subgame of 88 plies and its many clearly illegal positions and shallow wins. Encouraged by this and ever the optimist, Haworth sent requests for 4Q to Nalimov and Wirth, both known to be leading, active contributors in this field.

Both responded quickly to the moment and to the challenge, agreeing to make any results publicly available. To everyone's surprise except their own, they produced self-consistent 4Q EGTs within days which, under all tests, confirmed each other and Stiller's results. Each chose to strength-test their code, producing almost incidentally the tables for KNNKNN, KRRKRR and KBBKBB as well.

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Nalimov optimised *depth to mate* (DTM) and Wirth optimised *depth to force-conversion* (DTC), adding to the degree of independence between the two sets of results. The World Team supported and implemented the authors' *pro bono publico* principles by publicising the existence of the 4Q EGTs and making them freely available. This was done via the game's bulletin board, the WWW Computer Chess Club (1999) and a public ftp site (Hyatt, 1999) which soon sported an upgraded version of CRAFTY exploiting 6-man EGTs.

Several on the World Team downloaded both the new CRAFTY engine and the 4Q EGT. Kasparov had done the same and this raised the prospect of the game ending with perfect 4Q play. Certainly his official website showed Black dramatically securing a 4Q draw which was both immaculate and bizarre, Black sacrificing both Queens to force stalemate: see Appendix B4.2. Two 4Q-services were set up on the web (Mobley, 1999; Tamplin, 1999) and these added to the unusually significant contribution that EGTs were already making to the analysis of the game.

The production of 4Q on request was notable in itself and re-opened the 6-man chapter of endgame history. It was also the first time that two authors had worked simultaneously on the same endgame. The event suggested a comparative study to Haworth. The following sections therefore describe the 6-man EGT challenge, the two approaches and the sets of results. The paper includes a survey of the concepts involved to suggest some nomenclature and principles for EGT generation in the future.

2. THE 6-MAN ENDGAME DOMAIN

A complete EGT must include the value and depth of all legal positions in its scope. It may also cover unreachable positions whose illegality its generator program has not discovered, see Table 1 which includes all positions cited in this paper. The positions are Gödel-numbered using a 1-1 function *Index(Pos)* that must have an inverse in *Pos(Index)* if a position is to be easily found from its index.

The next sections introduce the key concepts of *notionally considered*, *indexed*, *legal* and *broken positions*.

2.1 Considered Positions

The simplest approach to creating *Index(Pos)* is to number the chessboard squares 0-63, define an order for the n men, say [wK, bK, wQ, ...], list the squares $\{s_i \mid i \in [0, 63]\}$ of the men for position *Pos* and define *Index*(*Pos*) = $64^n.\kappa + \sum 64^i \times s_i$ where $\kappa = 0$ or 1 for wtm or btm positions respectively.

In effect, the chessboard gives rise to a *hyperboard* or *n-cube* with edges of length 64 where each chess position occupies its own cell. This indexing scheme sufficed for 5-man EGTs (Thompson, 1986; Edwards *et al.*, 1995). For 6-man tables, there are 64^6 or 68,719,476,736 cells and this number is not conveniently manageable by an EGT generator program accessing a 4 Gigabyte address space, the maximum in a 32-bit architecture.

However, many of these positions are equivalent and/or illegal. The required index range may therefore be reduced by indexing only positions in some standard form and by avoiding as many of the illegal ones as possible. This is done by first ensuring that all indexed positions satisfy condition C as defined below:

- C_1 legality — there is at most one piece on any square
- C_{KK} legality — the two Kings are not next to each other
- C_L legality — $C_1 \wedge C_{KK}$
- C_{ad} symmetry — a specified K (e.g., the K of the side to move, stm) must be on one of files a-d
- C_C symmetry — if Wh. and Bl. forces are equal, the position is replaced by its btm equivalent
- C_P existence — Pawns are present but only on ranks 2-7
- C_8 symmetry — the specified K is in a specified a-d octant (here, a1-d1-d4)
- C_D existence — the specified K is on the diagonal of the specified octant (here, a1-d4)
- C_{TE} symmetry — the other K is in the *full-board triangular extension* of the chosen octant (here, if the specified King is on a1-d4, the other King is in a1-h1-h8)
- C_S symmetry — $C_{ad} \wedge C_C \wedge \{ C_P \vee [C_8 \wedge (\neg C_D \vee C_{TE})] \}$
- C composite — $C_L \wedge C_S$

Key	4Q?	Position	stm	Val.	DTM	DTC	DTF	Notes
					ply	ply	ply	
Maximal Positions								see Appendix B1
M1	yes	7K/3q4/8/3q4/8/7Q/1k5Q/8	b	1-0	100	88	88	the maxDTM/DTC btm 4Q position
M2	yes	7K/3q4/8/3q4/8/7Q/7Q/1k6	w	1-0	99	87	87	maxDTM/DTC wtm 4Q position
M3	yes	8/4q3/8/6k1/2Q5/2K5/Q7/7q	w	1-0	99	81	81	maxDTM wtm 4Q position
M4	no	8/q7/P6k/Q7/8/8/8/6K1	w	1-0	235	213	141	maxDTF wtm KQPKQ position .. KQP(a6)KQ
M5	no	8/2Q4K/8/1q6/8/3P4/8/7k	w	1-0	247	227	?	maxDTM/DTC wtm KQPKQ position .. KQP(d3)KQ
Zugzwangs: all wtm draws								see Appendix B2: note when bQ is captured in M-M play
Z1	yes	8/K1Q5/7q/1q6/8/Q7/8/1k6	b	1-0	24	18	18	bQ captured on ply 4
Z2	yes	8/8/1q6/8/Q1Q5/8/1k2K3/1q6	b	1-0	20	6	6	bQ captured on ply 2
Z3	yes	8/q7/5q2/8/8/8/6QQ/k6K	b	1-0	22	6	6	bQ captured on ply 4; see Appendix B5.1 for the derived study
Z4	yes	Q7/8/8/5q2/7q/K3Q3/8/3k4	b	1-0	28	18	18	bQ captured on ply 14
Z5	yes	8/8/8/8/1Q2Q3/8/5q2/1K1k1q2	b	1-0	24	10	10	bQ also captured here on ply 10
Z6	yes	8/8/1Q6/1Q3K2/8/8/8/q1k1q3	b	1-0	30	20	20	bQ captured on ply 14
Z7	yes	8/q7/8/8/2Q5/q7/1k2K1Q1/8	b	1-0	26	20	20	bQ only captured on ply 22; n.b. White's quiet move 2
Z8	yes	8/8/5q2/8/Q7/2k3KQ/1q6/8	b	1-0	40	28	28	bQ captured on ply 22
Legal Positions								
L1	yes	1Q6/Q1k5/8/2K5/8/8/3qq3/8	b	1-0	0	0	0	Prior move: 1.b8=Q# ... double check by Queens is possible
L2	yes	QQ1k4/6qq/3K4/8/8/8/8/8	b	1-0	0	0	0	Prior move: 1.axb8=Q# or 1.Qxb8#
Illegal Positions								🖰 ==> rated 'illegal' by EN (DTM) or CW (DTC)
I1	no	8/8/8/8/8/8/P1K5/k7	w	'1-0'	23	11	1	KPK: no anterior move (Haworth and Velliste, 1998)
I2	no	8/8/8/K7/8/2k5/1P6/8	w	'1-0'	🖰	15	1	KPK: 1. ... Kc3 was illegal (Haworth and Velliste, 1998)
I3	no	8/8/8/8/8/8/n3n2Q/k1K5	w	'1-0'	21	9	9	KQKNN: impossible double check by two Knights
I4	yes	8/1Q6/Q1k5/8/2K5/8/8/3qq3	b	'1-0'	0	0	0	Impossible double check by two Queens (cf. L1 above)
I5	no	8/8/8/8/8/8/K1p5/Qbk5	w	'1-0'	29	17	17	KQKBP: no move before the last 1. ... b1=B (Lippold, 1997)
I6	no	8/pPp1p1p1/8/k1K4/8/B7/8/B7	w	'1-0'	3	1	1	Impossible Wh. *promoted force* B: 1. b8=Q any 2. Qbx#'
I7	yes	1Q6/Q1k5/8/2K5/8/8/8/2qq4	b	----	🖰	🖰	🖰	Wh. (sntm) in check; Bl. cannot move
I8	yes	8/1Q6/Q1k5/8/8/2K5/8/2qq4	b	----	🖰	🖰	🖰	Wh. (sntm) in check; Bl. has a move
Both Kings on the long diagonal								
S1	yes	7q/6q1/8/8/3QQ3/2K5/8/k7	w	1-0	5	2	2	S1' = 7q/6q1/8/3Q4/3Q4/2K5/8/k7; n.b. 1. Kb3+ wins
S2	yes	7q/6q1/8/3Q4/5Q2/2K5/8/k7	w	=	=	=	=	S2' = 7q/6q1/3Q4/8/4Q3/2K5/8/k7
S3	yes	8/8/5qq1/4Q3/3Q4/2K5/8/k7	w	1-0	3	1	1	S3' = 8/5q2/5q2/4Q3/3Q4/2K5/8/k7; 1. Qxg6 wins
S4	yes	5q2/7q/8/4Q3/3Q4/2K5/8/k7	w	1-0	3	3	3	S4' = 6q1/8/7q/4Q3/3Q4/2K5/8/k7
The Kasparov-World Game and Analysis								see Appendix B3
GK1	no	7Q/1p6/3p2K1/6P1/8/8/8/1k1q4	w	?	?	?	?	After 50. .. d1=Q: game enters KQPKQPP
GK2	no	8/8/3p1K2/6P1/1Q6/8/8/k2q4	b	1-0	164	?	?	After 55. Qxb4: game enters KQPKQP
GK3	no	8/6K1/6P1/3p4/3Q4/5q2/8/1k6	b	1-0	158	?	?	After 58. g6: Bl. played 58. ... Qe4? [Qf5']
AN1	no	4q3/8/6P1/7K/5Q2/8/2k5/8	w	1-0	159	131	?	1. Qd4" is the only winning move
Weaknesses of pure *F* and *M* strategies								
RR	no	8/8/8/8/6KR/6R1/k7/8	w	1-0	3	2	2	'M-M': Rh2+ Kx1 2. Rg1#. 'F-M': 1. Ra3 Kxa3 & mt in 17 ply
BN1	no	6Q1/8/8/1n6/8/7K/7B/1k5B	w	1-0	5	2	2	'M-M': 1. Qb3+' Ka1' 2. Be5+' Nc3 3. Bxc3#'
BN2	no	8/8/8/1n6/8/7K/k6B/7B	w	1-0	127	103	103	From BN1, 'F-M': 1. Qa2+ Kxa2 [BN2]
NP1	no	6N1/8/7p/8/8/8/3N1k2/7K	w	1-0	229	228	?	maxDTM KNNKP(h) pos. (Dekker, 1989): M = C = MC = CM
NP2	no	8/8/4k3/8/7p/6KN/6N1/8	w	1-0	181	180	?	After 24. .. h4: h3 must be forced before m75.
NP3	no	8/8/8/8/7p/1N1K4/7N/4k3	w	1-0	105	104	≤ 24?	After 62. ... Ke1: Wh. needs Strat F; C = M ==> Nd2
NP4	no	8/8/8/8/7p/3K4/3N3N/4k3	b	'1-0'	104	103	35	After 63. Nd2: 63. .. Kf2 forces h3 to at least m80 ... draw!
Depth-illustrating positions								
NP5	no	7k/5K2/8/4N1N1/8/8/7p/8	b	1-0	2	1	1	Bl. starts & finishes this; "Conversion in 0" in Wh. moves

Table 1: Chess positions[4] cited in this paper.

The condition C_L is necessary but not sufficient to ensure the legality of a position. An endgame generator must check that the positions it generates satisfy C_L or mark them as *broken*, see below, and then reflect and rotate them to satisfy C_S. Legality is further tested during the initial phase of the EGT generation process. This set of *considered positions* was a common, notional starting point for Stiller, Nalimov and Wirth.

[4] DTF is *Distance to FIDE event* where *FIDE event* is defined here as P-push, capture or mate.

The set of legal K-K positions in Pawnless- or Pawn-endgames can be listed and counted as follows:

N_K = |{K-K positions | C}| = $33 + 3 \times (30 + 58 + 55) = 462$
N_{KP} = |{K-K positions | $C_L \wedge C_{ad}$ }| = $4 \times 60 + 24 \times 58 + 36 \times 55 = 1806 \approx 3.91 \times N_K$

The explicit management of these positions in a table generator ensures that $C_{KK} \wedge C_{ad}$ is satisfied and also replaces the index's *n-cube* with an *n-space* of less volume and lower dimensionality. The index ranges required for these illustrative 6-man endings can be calculated in the simplest way as:[5]

N_a = $N_K \times 62 \times 61 \times 60 \times 59 \times 2$ = |{wtm/btm KQRKBN positions satisfying C}| = 12,370,770,720
N_b = $N_a / 2$ = |{btm KQRKQR positions satisfying C}| = 6,185,385,360
N_{aP} = $N_{KP} \times 48 \times 61 \times 60 \times 59 \times 2$ = |{wtm/btm KQRKQP positions satisfying C}| = 37,438,813,440
N_{bP} = $N_{aP} / 2$ = |{btm KQPKQP positions satisfying C}| = 18,134,425,260

N_b and N_{bP} are only 9.00% and 26.39% of 64^6 respectively, a considerable improvement. The next section introduces three ideas which further reduce the size of the index ranges to be managed.

2.2 Indexed positions

Further rules which may be exploited to ease the task of table generation are:

- C_{LM} symmetry — *L* like men of a side are indexed as a set, reducing the index range by a factor of *L!*
- C_{UC} legality — a side-to-move man, Q/R/B/N/P, may not give an *unblockable* check to the sntm K

Nalimov and Wirth, but not Stiller, used C_{LM} to reduce the number of distinct 4Q positions to 1,546,346,340, just 12.5% of N_a. The function *Index(Pos)* must calculate the sequence number of a specific arrangement of the *L* like men and *Pos(Index)* requires a function to calculate the specific arrangement of the *L* like men for a specific index.

Suppose now that btm positions are being generated and the Kings have been placed. Depending on the position of the white King, condition C_{UC} constrains a black Queen from being on 3, 5 or 8 squares. Similar calculations can be done for the R, B, N and P: those for N and P also involve the position of the black King. Condition C_{UC} in effect replaces the one *n*-space index by a number of *n*-space sub-indexes, each tailored to Black's force profile and the positions of the white King and sometimes the black King. The same argument holds of course for wtm positions. It is then necessary to list the starting point and dimensions of each sub-index *n*-space in a metatable.

It is always possible to partition an endgame according to the positions of one or more men (Lake, Schaeffer and Lu, 1994) and particularly convenient to do so in P-endings such as KQPKQP which was significant to the outcome of this game. Positions with the same pawn formation may be considered as a sub-endgame of a P-endgame and their part of the EGT can be managed as a *logical block* split into two blocks by condition C_{ad}. This has a runtime advantage for chess engines; as Steinitz said, Pawns cannot move backwards so certain positions will be unreachable. References to the EGT will be clustered in the remaining subgames which are most relevant.

2.3 Legal Positions

Determining whether a position is legal or not can be difficult (Lippold, 1997) as the following positions from Table 1 illustrate. Double check by two Queens is possible and L1-L2 are in fact legal. Positions I1-I4 have no prior move, I4 being merely L1 shifted down one row. Lippold's I5 has no move before the previous 1. ... b1=B. I6 has the impossible *promoted force* of a second black-square Bishop, a fact better proved than discovered by a retro-search that can only be contemplated. I7-I8 have the side not to move, *sntm*, in check.

EGT-generating programs usually do not root out all illegal positions and there is no need for them to do so. Table generators will spot 'side not to move in check' but I1-I6 are typically regarded as being legal. The EGT's prime purpose is to provide the value and depth of all legal positions. Derived statistics will be affected by illegal positions being treated as legal but this is a secondary consideration.

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[5] For further simplicity, these calculations ignore the fact that the coding of any *en passant*, if not castling, rights is now customarily included (Heinz, 1999; Wirth and Nievergelt, 1999; Nalimov and Heinz, 2000).

2.4 Broken Positions

As discussed above, an EGT generator cannot typically classify positions as *legal* and *illegal*. It can only mark some positions as *seen to be illegal* and the term *broken* was therefore coined (Edwards *et al*., 1995) for these. In addition, the *broken* classification allows a generator to remove from consideration those positions which are *redundant* or do not satisfy condition C above.

A position with both Kings on the a1-h8 diagonal can satisfy condition C before and after being reflected in that diagonal: the EGT may therefore represent two equivalent positions. Positions S1 to S4 of Table 1 illustrate that the best way to remove one is to mark the higher-indexed position as *broken*: there is no simpler rule in terms of a sequence of nominated pieces' positions. This does not however remove the requirement on the generator to standardise any such positions to the chosen form (Van den Herik and Herschberg, 1985) if one has been defined.

With one version of such positions marked as *broken*, each equivalence class of positions is represented just once. EGT statistics then guarantee to represent distinct positions and the subsequent work of generating the EGT is reduced by some 4.35% (~21/483).

3. SUMMARY OF ALGORITHMS

This section documents the common structure of Nalimov's and Wirth's algorithms and the next sections detail the differences at points marked * between them. The common principle is that deeper wins are identified from the shallower wins of their successors. Val[i] becomes the endgame table, holding the value and depth of Position(i). The function *Evaluate* sets *ChangeFlag* to *true* if the depth of any position is changed in array *Val*.

{initialise *ChangeFlag* and endgame table *Val*} ChangeFlag ← *true*;
    determine* index range R required for wtm and btm positions satisfying $C \wedge C_{LM}$;
    for i ∈ [1, R] **do** Val[i] ← *broken* **end_for**;
    for each Position(i) **do if** Position(i) is not *broken* **then** Val[i] ← InitialValue* **end_if end_for**;
{seed *Val* with the value and depth of terminal won/lost positions in the endgame chosen}
    for each unbroken Position(i) **do**
        if side-to-move is mated **then** Val[i] ← 0 **end_if**;
        if side-to-move can convert to win **then** Val[i] ← Depth1* **end_if**;
        if side-to-move must convert to lose **then** Val[i] ← Depth2* **end_if end_for**;
{iterative search*}
    while ChangeFlag = *true* **do** ChangeFlag ← *false*;
        for each necessary* *i* **do** Evaluate*(Val[i], {Val[j] | Position(j) is a neighbour of Position(i)}, ChangeFlag);
        end_for end_while;
{check self-consistency of endgame table *Val*}
    for each unbroken Position(i) **do**
        Compatibility-Check(Val[i], {Val[j] | Position(j) is a successor of Position(i)}) **end_for**;
{finish - endgame table *Val* completed}

3.1 Nalimov's algorithm

Nalimov introduced condition C_{UC} to endgame indexing, namely the avoidance of *unblockable* checks by the side to move, *stm*, as described in section 2.2. Such checks cannot be blocked by placing further men on the board. Nalimov and Heinz (2000) describe the general use of condition C_{UC} in more detail. Here, the constraints on the Queens save some 19.98% of the index space because:

$$R = \{33 \times 59 \times 58 + [58 \times 57 \times 56 + (30 + 55) \times 54 \times 53] \times 3\} \times (60 \times 59) / (2! \times 2!) = 1{,}237{,}357{,}440$$

Unless the side-not-to-move is in check, Val[i] is initialised to *draw*. Both versions of unbroken positions with both Kings on a1-h8 are retained.

The depth of a position about to be converted and won, *Depth1*, is set to the converted position's DTM + 1. The depth of a position which must be converted but is lost, *Depth2*, is set to the converted position's DTM.

Nalimov's iterative process is a sequence of *forward searches*. It is necessary to visit each unbroken Position(i) in each search and *Evaluate* infers or re-infers its value and depth from those of its *neighbours*, i.e., its successors. A position is rated an stm win if any successor has been designated a loss for the sntm. Similarly, a position is rated an stm loss if all sntm successors are wins for the sntm. Depths in moves are eventually ascribed to any position which is a win for White or Black. The search making no value/depth changes terminates the process.

3.2 Wirth's algorithm

Wirth does not use C_{UC} but calculates R more simply as $462 \times 62!/(58! \times 4! \times 2! \times 2!)$. The higher-indexed version of a position with both Kings on the a1-h8 diagonal is marked as *broken* as are positions with the sntm K in check. The *InitialValue* of Val[i] is the number of successor positions in the target endgame profile, here KQQKQQ. Positions which can be converted to winning positions or must be converted to lost positions have *Depth* set to 1 ply.

Wirth used Gasser's (1995, 1996) domain-independent RETROENGINE which executes an iterative process of retro-searches. This minimises the work involved by two techniques (cf. Thompson, 1986). First, the *necessary positions* are only those whose information changed in the last iteration. Second, the algorithm counts down the number of successors of a position which have not yet been proved to lose. The *neighbours* of a position here are its predecessors and the function *Evaluate* changes their Val[i].

Positions which are stm losses back up, via an *unmove generator* which cannot introduce new men by *uncapture*, to a set of sntm wins. However, stm wins merely reduce their predecessors' number of unresolved successors by one, and only if this number is zero is the predecessor position marked as *lost in n plies*. The losing side is of course trying to avoid mate rather than helping to get itself mated.

Evaluate assigns any positions resolved as wins or losses to the set of *necessary positions* for the next iteration.

4. THE RESULTS

The summary figures are in Table 2; the per-level detail is in Table 3. Derived %-statistics differ slightly because of the different treatments of *indexed* and *broken* positions.

			Nalimov: DTM		Wirth: DTC	
Type of KQQKQQ Position		**%**	**# of positions**	**%**	**# of positions**	**%**
Notionally considered	NC		1,546,346,340		1,546,346,340	
Broken: not included in the index	BA	BA/NC	308,988,900	19.98%	0	0.00%
Indexed	I	I/NC	1,237,357,440	80.02%	1,546,346,340	100.00%
Wins by White (to move)	W	W/U	417,439,889	61.10%	407,438,945	61.07%
Draws	D	D/U	129,060,182	18.89%	125,942,240	18.88%
Losses by White	L	L/U	136,739,434	20.01%	133,753,410	20.05%
Broken: found after indexed	BB	BB/I	554,117,935	44.78%	879,211,745	56.86%
Unbroken: W + D + L	U	U/NC	683,239,505	44.18%	667,134,595	43.14%
Total *broken*: BA + BB	TB	TB/NC	863,106,835	55.82%	879,211,745	56.86%
Zugzwangs			8		8	
Maximal White wins	wtm		2	99 ply	1	87 ply
	btm		1	100 ply	1	88 ply

Table 2: Summary of comparative KQQKQQ statistics.

Nalimov's DTM results						Wirth's DTC results					
Wh. to move			Bl. to move			Wh. to move			Bl. to move		
ply	**# pos.**	**H%**	**ply**	**# pos.**	**H%**	**ply**	**# pos.**	**H%**	**ply**	**# pos.**	**H%**
---	---	81.11	0	12,437,439	80.76	---	---	81.12	0	12,175,059	80.77
1	39,998,840	79.54	2	16,579,504	78.99	1	316,072,316	62.84	2	57,812,520	55.19
3	49,991,811	77.13	4	17,758,848	76.38	3	37,282,346	48.34	4	22,508,278	43.09
5	51,229,915	73.94	6	16,695,951	73.03	5	20,089,584	37.40	6	13,726,945	32.82
7	50,758,262	69.83	8	14,597,887	68.77	7	12,776,721	27.91	8	9,641,063	23.69
9	36,466,693	65.74	10	10,881,463	64.72	9	7,843,787	19.89	10	6,278,943	16.55
11	25,052,524	62.13	12	7,800,984	61.24	11	4,697,304	13.87	12	3,940,787	11.49
13	20,927,028	58.64	14	5,902,029	57.85	13	2,819,439	9.70	14	2,407,002	8.11
15	22,279,272	54.54	16	5,321,561	53.67	15	1,770,713	6.91	16	1,555,414	5.83
17	27,517,414	48.59	18	5,710,598	47.39	17	1,173,016	4.99	18	1,053,375	4.23
19	29,211,870	40.28	20	5,948,697	38.59	19	808,989	3.64	20	734,835	3.10
21	23,383,829	30.91	22	4,966,445	29.02	21	573,039	2.67	22	523,000	2.27
23	15,457,652	22.42	24	3,524,914	20.74	23	409,808	1.96	24	375,661	1.67
25	9,577,184	15.79	26	2,424,901	14.44	25	300,273	1.44	26	277,253	1.23
27	5,749,963	11.05	28	1,679,023	10.00	27	222,312	1.05	28	204,020	0.90
29	3,456,024	7.78	30	1,188,714	6.99	29	162,783	0.77	30	149,055	0.65
31	2,133,377	5.54	32	864,955	4.94	31	119,817	0.56	32	107,528	0.47
33	1,360,191	3.98	34	636,719	3.52	33	87,100	0.41	34	79,614	0.34
35	896,305	2.87	36	474,924	2.52	35	63,662	0.29	36	56,479	0.25
37	598,874	2.08	38	355,967	1.81	37	45,061	0.21	38	40,019	0.18
39	411,875	1.50	40	262,732	1.30	39	32,811	0.15	40	29,504	0.13
41	288,680	1.09	42	192,389	0.94	41	24,296	0.11	42	21,896	0.09
43	200,864	0.79	44	141,737	0.68	43	17,751	0.08	44	15,662	0.07
45	140,579	0.57	46	105,073	0.49	45	12,471	0.06	46	11,440	0.05
47	98,812	0.42	48	78,693	0.35	47	8,890	0.04	48	8,008	0.04
49	70,929	0.30	50	56,693	0.26	49	6,184	0.03	50	5,256	0.03
51	47,831	0.22	52	39,007	0.19	51	4,528	0.02	52	3,594	0.02
53	34,250	0.16	54	28,044	0.14	53	3,130	0.02	54	2,641	0.02
55	25,618	0.12	56	22,062	0.10	55	2,196	0.01	56	1,795	0.01
57	20,122	0.09	58	16,204	0.08	57	1,618	0.01	58	1,567	0.01
59	14,053	0.07	60	12,097	0.06	59	1,473	0.01	60	1,234	0.01
61	10,568	0.05	62	8,910	0.04	61	1,342	0.01	62	1,039	0.01
63	7,286	0.04	64	6,588	0.03	63	1,284	0.00	64	788	0.00
65	5,206	0.03	66	4,815	0.02	65	943	0.00	66	686	0.00
67	3,949	0.02	68	3,347	0.02	67	769	0.00	68	494	0.00
69	2,680	0.01	70	2,152	0.01	69	470	0.00	70	309	0.00
71	2,024	0.01	72	1,730	0.01	71	306	0.00	72	254	0.00
73	1,669	0.01	74	1,190	0.01	73	140	0.00	74	104	0.00
75	1,336	0.01	76	1,123	0.01	75	72	0.00	76	102	0.00
77	1,095	0.01	78	790	0.00	77	65	0.00	78	69	0.00
79	935	0.00	80	686	0.00	79	43	0.00	80	53	0.00
81	776	0.00	82	519	0.00	81	52	0.00	82	45	0.00
83	511	0.00	84	421	0.00	83	32	0.00	84	17	0.00
85	472	0.00	86	238	0.00	85	8	0.00	86	2	0.00
87	267	0.00	88	224	0.00	87	1	0.00	88	1	0.00
89	148	0.00	90	130	0.00	89	0	0.00	90	0	0.00
91	106	0.00	92	125	0.00	91	---	---	92	---	---
93	104	0.00	94	117	0.00	93	---	---	94	---	---
95	92	0.00	96	62	0.00	95	---	---	96	---	---
97	22	0.00	98	11	0.00	97	---	---	98	---	---
99	2	0.00	100	2	0.00	99	---	---	100	---	---
101	0	0.00	102	0	0.00	101	---	---	102	---	---

Table 3: Detailed KQQKQQ statistics - White wins in *n* plies.

It is said (Beasley and Whitworth, 1996, p. 180) that "more complicated positions may be evaluated by ignoring pairs of like pieces". In fact KQKQ, see Table 5 in Appendix A, has only 41.74% wins and 0.45% losses for the first player. The figures and comparison are affected by shallow wins which are the inevitable result of active play continuing from the previous phase. If pieces are not actually left *en prise*, there is typically the opportunity for a fork, skewer, discovered check or other equally sharp, winning tactic.

In Table 3, the column *H* shows the percentage of positions which are in fact won beyond the horizon of a search-engine armed only with 5-man EGTs. The engine may have a full-width search capability of *h* plies but may only be able to search *q* plies into the 4Q domain. For example, 37.40% of positions unresolved by a 5-ply search into 4Q are won or lost. This is of course different from the density of wins/losses of greater than *q* plies in the set of all *unbroken* positions, or the percentage of wins greater than 5 plies.

Nalimov's one maxDTM btm loss in Table 2 equates to the '2' 100-ply losses in Table 3 which mirror each other.

5. THE COMPUTATIONS

Lewis Stiller's innovative computation mapped the structure of the EG position set to that of the SIMD (Single Instruction, Multiple Data) Connection Machines, CM-2 and CM-200 (Hillis, 1985). These each had 64k processors running in step at 7MHz and 10MHz respectively. Nalimov created 4Q on a 2GB, 500MHz server, working completely 'in store' and avoiding all disc thrashing. The EGT was completed and checked for self-consistency successfully in two days. Wirth used a 1GB, 450MHz G3 PowerMac and sustained a total of approximately 100GB of disc traffic; the computation and integrity-check took five days.

These were colossal number-crunching feats by any standards but computations of this complexity are open to systematic or sporadic errors affecting hardware or software. Cosmic rays, power and electromechanical faults, flawed systems software and human error have all played their part in the past (Gasser, 1995; Wirth and Nievergelt, 1999) although Nalimov reports that no hardware errors have ever occurred on the servers he has used. As the results are not self-evidently correct in themselves, an independent integrity test to underpin blind faith in correctness is vital.

Both EGT authors therefore confirmed that the value and depth of each position was consistent with that of its successors. This was done by a parallelisable forward pass of the EGTs checking, for each position, that:

- a position won by the stm in p plies is succeeded by positions lost in $\leq p\text{-}1$ plies,
- a position lost by the stm in p plies is succeeded only by positions won in $\geq p\text{-}1$ plies.

This test ensures that any *unmove* generator is the exact inverse, within the target profile, of the *move* generator, and that no random errors have occurred. It cannot check any software which is common to itself and the generation process. Both authors also first strength-tested their '5-man' codes in the 6-man domain on the EGs KNNKNN and KRRKRR. Nalimov's code had also produced DTM EGTs which agreed with previous DTM EGTs (Edwards *et al.*, 1995). Each reproduction of a previous Stiller result suggested that all three EGTs were correct and aligned.

6. ENDGAME SERVICES

Nalimov provides practical tables for chess engines and these are now widely used by many top programs including 11 of the 30 WCCC '99 participants (Beal, 1999; Feist, 1999). Code, data and essential statistics are all available via Hyatt's public ftp site. Two www services (Mobley, 1999; Tamplin, 1999) were set up to help those users who did not wish to actually download the EGTs themselves.

Tamplin's established service provides all available moves for a position, indicating whether they are optimal, value-preserving or whether they lose a half or full point. Mobley's service, now discontinued, allowed one piece to be in *free position* and returned the set of matching positions in the EGT with their values. Wirth distributed his EGT-access software and 4Q EGT to Haworth and Marko. It provided reassuring second-sourcing of Nalimov's position evaluations.

7. OBSERVATIONS ON EGT GENERATION

The approaches of Nalimov, Wirth and others provide concepts and guidelines for future work.

Goal	GV	GF	GC	GM	GR
Goal Description	Game value at least position value	Mate, Capture or P-push: i.e. *FIDE event*	Mate, Capture or P-conversion	Mate	Mate within the FIDE 50-move rule
Minimax strategy	V	F	C	M	R
Metric	position value	DTF	DTC	DTM	DTR = DTF
Position depth	{+1, 0, -1}	d_f ply	d_c ply	d_m ply	$d_r = d_f$ ply
Endgame Table	EV	EF	EC	EM	ER
Example EG tables by:					
Komissarchik & Futer (1974)	none	KQPKQ	none	none	none
Arlazarov & Futer (1979)	none	none	KRPKR	none	none
Thompson (1986, pp. 133/5)	none	KxPKx, x = Q/R	KQKxx, x = B/N ...	KQNKQ ...	3- and 4-man
KT: CD & Tamplin (1999)	none	none	5-man	3- & 4-man	3- and 4-man
Lake et al. (1994) for checkers	2- to 8-man	n/a	n/a	n/a	n/a
Edwards (1995)	none	none	none	any done	3- and 4-man
Nalimov (2000)	none	none	none	3-, 4- & 5-man	3- and 4-man
Nalimov (2000)	none	none	none	10 KxxKyy	none
Wirth (1999)	none	none	KPPKP, KQQKQQ	3- & 4-man	3- and 4-man

Table 4: Goal-oriented strategies, metrics, depths and EG tables.

The authors chose different metrics in DTC and DTM. Both have their advantages and supporters so it is perhaps not possible to focus on one metric for second-sourcing purposes. Note, e.g., from positions M1-M5 of Table 1, that the choice of metric affects the set of maximum-depth positions. Table 4 lists the various goals that might in general be chosen and reviews the endgame work done to date. It discriminates between the related concepts of *Goal*, *Strategy*, *Metric*, *Position Depth* and *Endgame Table*. Its notation facilitates an examination of strategy-specific play; in this paper, for example, *M-F* indicates White using strategy *M*, Black using strategy *F*.

As Nalimov produces tables to support play over the board, the issue of the FIDE 50-move rule (cf. Dekker, Van den Herik and Herschberg, 1989) has to be discussed. The positions M4, BN1-2 and NP1-4 of Table 1 are salutary warnings that strategies M, C and F alone cannot win all positions that can be won in the context of the rule.

Because KQQKQQ is won in exactly 50 moves, it is not necessary to consider here the production of tables EF and ER. Also because sufficient memory was available, it was not necessary to produce EV tables to support goal GV for KQQKQQ although there is an argument that such tables are useful for chess engines playing in real time.

Both Nalimov's and Wirth's EG tables provide the value - win, draw or loss - and depth of both wtm and btm positions. This is important for two reasons. First, chess engines should not have to search forward one ply from btm positions (Heinz, 1999). Second, although Black is traditionally the weaker side, 6-man endings are showing a higher percentage of wins for Black than previous endings.

If an EM table indicates "side to move wins in *m* moves", it is possible to infer that this is *2m-1* plies, assuming the convention is being followed of quoting depth in the moves of the side-to-move. However, if table EC analogously indicates "side to move wins after conversion in *m* moves", it is not possible to infer whether this is *2m-1* or *2m* plies as position NP5 indicates. The loser may be forced to or wish to make the conversion (cf. Thompson, 1990).

Nalimov's use of advanced index schemes and compression techniques contribute to the use of his tables in actual play. Work currently in progress, including that on KQPKQP (Karrer, 1999), shows that there are further ways of optimising the runtime performance of these tables. Index ranges may in some cases be reduced further and there are major gains to be had by partitioning EG tables for P-endgames. This means that although P-endgames cannot be produced definitively before the pawnless endgames have been done, their tables need not be the largest that chess engines have to manage in real time.

The plethora of EG tables now available suggests that standards should perhaps be defined for the provision of these results to chess engines and humans both for play and analysis. A stable, high-level interface between chess engines and endgame services is desirable, the engine merely presenting the position and a definition of the information required. Behind the interface, the endgame service would find the EG table or subtable of the appropriate type, manage the logistics of referencing these tables efficiently and minimise memory requirements.

Theorists also require access to endgame tables, preferably those not moderated by the 50-move rule, and to the statistics about them. Statistics can be published via the web in a way which facilitates the further analysis of the data by the end user.

8. A CHESS PERSPECTIVE

The creation of the 4Q EGT has had an effect on EGT progress and on the Kasparov-World game - and will have an effect on the game, art and science of chess in the future.

8.1 4Q and EGT progress

Stiller (1992, 1995, 1996) was the first to produce EGTs for six fully mobile men and it will clearly take some time before his 41 EGTs are recreated. Both Nalimov and Wirth tested their codes on the somewhat more easily generated KNNKNN and KRRKRR before attempting 4Q: both have lower *position fan out* and smaller maxDTM than 4Q. With the code proved, Nalimov went on to produce KBBKBB and all six KxxKyy ($x \neq y$; $x, y \neq P$) even though the latter feature twice as many positions.

Gasser's RETROENGINE, later developed by Wirth and Lincke, has solved Merrils, created Awari endgame tables to 28 stones, found the deepest 15-puzzle positions and computed the chess endings KPPKP, 4Q and KRRKBN. It is now working on KQRKQB and will no doubt be exercised on other chess endings.

With 4Q available, the World Team turned to the production of KQQKQP≈ and KQPKQP≈ tables (Karrer, 1999), the '≈' denoting the use of the simplifying but approximating *no underpromotions* heuristic. In the process, they considered the EV table but instead used endgame partitioning (Lake *et al*., 1994). Following Kasparov's win (Marko and Haworth, 1999), this work is helping to evaluate the game's last moves under the P=Q assumption.

8.2 4Q impact on the Kasparov-World game analysis

The Kasparov-World game was extraordinary from many points of view, not least that of the endgame expert. The very fact that EGTs played a significant part made it a rare event. A survey of Fatbase (Monkman, 1999) shows that only 164 games have arrived at any of the 3-3-man endgames with EGTs to date though many more would do so in background analysis. At one point, the World Team entertained the prospect of Kasparov having to win KQPKQP and KQQKQQ endgames against infallible play.

Table 1 shows three key endgame positions, GK1-3, and it is notable that GK1 without the two black Pawns is a draw. Black indeed sought to lose its Pawns safely and at one time over 120 lines of the World Team's analysis were simultaneously rated *theoretical draw* by the KQPKQ EGT. The 4Q, KQPKQ and KQKPP EGTs all helped to establish 52. ... Kc1 as best but, critically, this move did not win the vote. Around move 57, Ken Regan researched the safe KQPKQ positions of the game and discovered position AN1 - a 66-move loss to avoid. The 4Q EGT saved the World Team's computers from evaluating search trees with a potential branching factor of over 50. Peter Karrer reported after an experiment that the 4Q EGT was cutting CRAFTY's analysis time by 25%.

The preliminary analysis of the 4Q domain did not recommend a 4Q ending as a safe haven for Black. The fact that a draw might turn into a win for White merely by moving one piece one square made 4Q look like a particular dangerous minefield until it was explained that this is true for all endgames. Then the odds of a draw were examined, assuming Black dropped the game into a 'random' wtm 4Q position. The densities of first player wins, 61% with the *random* assumption and 76% assuming also that the second player cannot win, were not selling points.

Although Black would typically have had some freedom to improve the position before its P-promotion with checking moves, there was a danger that White could eliminate this freedom by some zwischenzug checks before its own P-promotion. Kasparov's views on this theme would be interesting. Since 4Q draws often involved a forced Q-exchange, the World Team aimed to force such a Q-exchange even before 4Q was reached.

For the future, it is inevitable that endgame tables will attract at least computer play towards guaranteed wins when these are identified by forward search. This phenomenon occurred in WCCC '99 (Feist, 1999, p. 152) when SHREDDER found and selected a 31-move win in KBBKN.

8.3 Mining KQQKQQ

Any new EGT should be mined for higher grade knowledge, both for its contribution to chess lore and for its verdict on past games, theory, problems and studies. Hopefully, some principles of good play (Haworth and Velliste, 1998) will be created or refined and some positions which surprise and delight (Wirth and Nievergelt, 1999) will emerge. A comparison of computer play with and without the EGT would indicate to what extent computers need the perfect information now available.

Only six games featuring 4Q have been recorded. Three were quick wins and two were unavoidable draws. In the remaining game, Black converted a drawn position to a 14-move win which White missed by accepting a draw on the next move, see Appendix B4.1. There are just two studies featuring 4Q in the main line, see Appendix B5. Of these, one (Elkies, 1993) was inspired by Stiller's discovery of the zugzwang position Z3 in Appendix B2.

The maximal depth positions have been identified and played out as is customary, even if they are in one sense at an extreme of and possibly not typical of the endgame domain as a whole. Appendix B1 lists the one btm and two wtm maxDTM White wins. Without perfect information, the win from position M3 with 16 'unique winning moves' for White would hang by a more slender thread than that from position M2. Both show the extraordinary ability of the two white Queens to combine as a team, ignoring even Black's Q-interpositions. This play currently looks like Michie's *Martian Chess* but will become more meaningful, especially in the shallower won positions.

No searches have been done to find maximal problems or studies (Lindner, 1991) satisfying various criteria but it is worth noting that retro-search, and RETROENGINE in particular, could be exploited to perform this work.

The set of mutual zugzwangs discovered by Stiller has been independently rediscovered, see Appendix B2. All are wtm draws and btm wins for White. They affect the optimal play of adjacent positions: it is an open question as to what their *basins of attraction* are, i.e., the four sets of positions from which optimal play in some metric necessarily reaches, can be forced by White or Black to reach, or may reach that zugzwang. Denoting these basins in a natural way by B_{min}, B_w, B_b and B_{max} respectively, they satisfy the set relations $\varnothing \subseteq \mathrm{B}_{\mathrm{min}} \subseteq \mathrm{B}_{\mathrm{x}} \subseteq \mathrm{B}_{\mathrm{max}}$, $x = w$ or b.

The 40 or so successive and precise Q-checks of the maximal mates raise the question of how common voluntary *quiet moves* are in forced wins: these are prized generally in the art of chess and seem particularly rare here. Mutual zugzwangs are reached of course with non-checking moves. The eventual winner typically rides out a checking sequence after a forced Q-sacrifice by the other side but these moves are neither voluntary or in 4Q. After Z7, White maintains the Royal Battery with the quiet 1. Qf2, the only winning move.

9. SUMMARY

The Kasparov-World game of 1999 was a new mode of play and a new computer contribution to the world of chess. It enabled a worldwide group on the web to work together to great effect on a shared problem against fixed, short-term deadlines. It demonstrated the depth of analysis achievable by the synergy of carbon and silicon intelligence in 'correspondence' situations, the value of perfect information in EGTs, and the current feasibility of attacking 6-man endgames, even 'on demand'.

Inspired by this event, the first two authors produced 4Q EGTs in two independent and entirely successful initiatives. Between them, they highlighted the range of decisions to be made while working in the EGT arena. This paper surveyed their approaches and results, deriving some guidelines for future workers in this field.

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Many people contributed to the work and the material in this paper. Lewis Stiller was most helpful in revisiting his 6-man results, both published and unpublished. Noam Elkies assisted with that history and with the analysis of Appendix B. Peter Marko, an outstanding volunteer ringmaster on the game's bulletin board, co-instigated the production of the 4Q EGT and the 4Q-services on the web. Robert Hyatt hosted Nalimov's 4Q EGT on his ftp server and promptly upgraded CRAFTY to use it. Peter Karrer provided software for a 4Q service hosted by Carter Mobley. John Tamplin extended his established EGT service and investigated several details here. Francis Monkman scanned Fatbase for 6-man endings, Noam Elkies and Harold van der Heijden contributed on '4Q' endgame studies and Stefan Meyer-Kahlen advised on the recent SHREDDER-REBEL game at WCCC '99.

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APPENDIX A: KQKQ STATISTICS

Nalimov's DTM results						Wirth's DTM results					
White wins in n ply						White wins in n ply					
Wh. to move			Bl. to move			Wh. to move			Bl. to move		
ply	# pos.	H%	ply	# pos.	H%	ply	# pos.	H%	ply	# pos.	H%
---	---	42.20	0	815	42.16	---	---	42.20	0	801	42.16
1	7,397	41.79	2	640	41.75	1	7,223	41.78	2	623	41.75
3	5,570	41.47	4	397	41.45	3	5,347	41.47	4	381	41.45
5	7,839	41.04	6	235	41.03	5	7,279	41.06	6	205	41.05
7	19,434	39.99	8	220	39.98	7	19,007	40.01	8	197	40.00
9	34,767	38.02	10	87	38.02	9	33,691	38.06	10	79	38.06
11	61,321	34.24	12	92	34.24	11	60,444	34.25	12	87	34.25
13	92,389	27.59	14	328	27.57	13	90,727	27.57	14	325	27.54
15	121,485	16.46	16	856	16.37	15	119,308	16.36	16	846	16.27
17	95,358	4.92	18	1,134	4.77	17	93,140	4.80	18	1,115	4.64
19	29,445	0.55	20	422	0.49	19	27,995	0.54	20	404	0.47
21	3,095	0.03	22	26	0.02	21	2,915	0.03	22	26	0.02
23	136	0.00	24	2	0.00	23	136	0.00	24	2	0.00
25	2	0.00	26	0	0.00	25	2	0.00	26	0	0.00
27	0	0.00	28	---	---	27	0	0.00	28	---	---
Wins	478,238	41.75	W, W/U	5,254			467,214	41.74	W, W/U	5,091	
Draws	662,096	57.80	D, D/U				646,911	57.80	D, D/U		
Losses	5,254	0.46	L, L/U				5,091	0.45	L, L/U		
Unbroken	1,145,588	73.26	U, U/I				1,119,216	64.05	U, U/I		
Broken	418,147	26.74	BB/I				628,068	35.95	BB/I		
Total broken	601,696		BA + BB				628,068		BA + BB		
Indexed	1,563,735		I				1,747,284		I		

Table 5: Detailed KQKQ statistics.

APPENDIX B: EXAMPLE KQQKQQ PLAY

Some notation has been introduced to indicate properties of the lattice of value-preserving moves:

' for *only optimal move*, " for *only value-preserving move*, ° for *only legal move*,
^{n} for *one of n unlisted equi-optimal moves*, [..., ...] for an equi-optimal move list and {...} for comments.

These additions complement established *Chess Informant* notation and are hopefully discrete enough not to reduce the readability of the moves themselves.

B1 The maxDTM and maxDTC wins for White

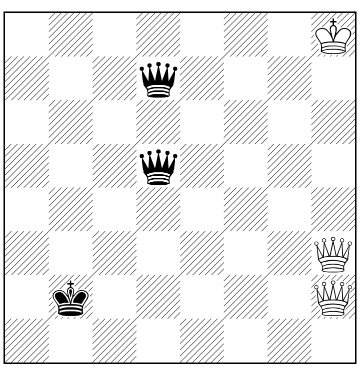
B1.1 Position **M1**: White mates in 100 plies. Stiller's optimal DTC line coincides with this chosen 'M-M' DTM-minimaxing line until they diverge with Black's 39th move.

1. ... Kb1' {pos. **M2**} 2. Qg1+" Ka2' 3. Qhh2+" Qd2' 4. Qg8+" Ka3' 5. Qa8+" Kb4 [Kb3] 6. Qhb8+" Kc4' 7. Qa6+' Kd5' 8. Qg8+ [Qb3+] Ke5' 9. Qg3+' Kd5' 10. Qf3+' Kc5' 11. Qf8+' Kd5' 12. Qaa8+' Kc4' 13. Qf1+' Kb3' 14. Qb8+' Kc3' 15. Qa1+' Kc4' 16. Qg8+' Kb4' 17. Qf8+' Q7d6' 18. Qb1+" Kc5' 19. Qg1+" Kb5' 20. Qe8+' Qd7' 21. Qe5+' Ka4' 22. Qga1+' Kb3' 23. Qg3+' Kc4' 24. Qf1+' Kc5 [Kd4] 25. Qa3+' Kd5' 26. Qb3+' Ke5' 27. Qb8+' Kd5' 28. Qf3+' Kc5' 29. Qa3+' Kd4 [Kd5] 30. Qb6+' Ke4' 31. Qg6+ [Qb1+] Kd4' 32. Qg1+' Ke4' 33. Qh1+' Kd4' 34. Qh4+' Kd5' 35. Qb3+' Ke5' 36. Qh5+' Kd4' 37. Qb6+' Ke4' 38. Qh1+' Ke5' 39. Qc5+' {and here the Stiller DTC line diverges from this DTM line} Q7d5' 40. Qe7+ [Qh5+] Kd4' 41. Qxd5+' {KQQKQ} Kxd5 42. Qd7+ [Qd8+] {skewer} Kc4 [Kc5] 43. Qxd2" {and mate on move 51}.

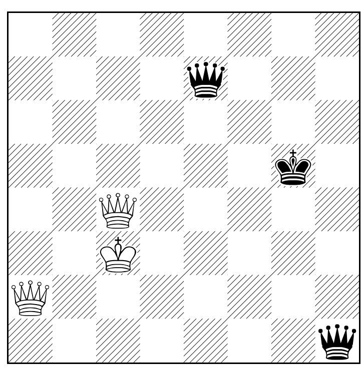
In the 40-move 4Q phase proper here, White has only one winning move 7 times and a two-way choice of optimals 3 times. Black has a two-way choice of optimals 3 times and interposes a Queen 4 times.

After White's m39, Stiller's DTC line continued:

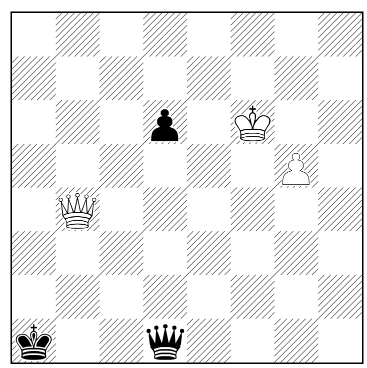
39. ... Q2d5' 40. Qh5+' Ke4' 41. Qh4+' Ke5' 42. Qe3+ [Qg5+] Kd6' 43. Qb6+ [Qb4+] Q5c6 [Q7c6] 44. Qf6+' Qe6 [Kd5] 45. Qxe6+ [Qxc6+] {KQQKQ} Kxe6° 46. Qxc6+" {and mate on move 53}.



M1: maxDTM btm 4Q Wh. win.



M3: maxDTM wtm 4Q Wh. win



GK2: GK-World after 55. Qxb4

B1.2 Position **M3**: White mates in 99 plies. Again, this is an 'M-M' line with play minimaxing DTM:

1. Qd2+" Kh5' 2. Qb5+" Kg4' 3. Qd4+' Qhe4' 4. Qe2+" Kf4' 5. Qdf2+" Kg5' 6. Qg3+" Kf6' 7. Qef2+" Ke6' 8. Qh3+" Kd5' 9. Qa2+" Kd6' 10. Qh6+' Kc7 [Kd7] 11. Qa7+' Kd8' 12. Qh8+' Qe8' 13. Qb6+ [Qb8+] Kd7' 14. Qg7+' Q8e7' 15. Qa7+' Kd8' 16. Qg8+' Qe8' 17. Qg5+' Q8e7' 18. Qga5+' Ke8' 19. Qh5+" Kd8' 20. Qb8+' Kd7° 21. Qd1+' Ke6' 22. Qb6+" Kf7' 23. Qh5+" Kf8' 24. Qb8+' Qe8' 25. Qh6+ [Qh8+] Kf7' 26. Qc7+' Q8e7' 27. Qh5+" Kf8' 28. Qc8+' Qe8' 29. Qcc5+' {and here the following DTC line diverges} Q8e7' 30. Qh8+' Kf7° 31. Qch5+' Ke6' 32. Qc8+" Kf6' 33. Qhh8+' Kg5' 34. Qcg8+" Qg6' 35. Qd5+" Qf5' 36. Qd2+' Qf4' 37. Qg8+' Kf5' 38. Qd3+' Qfe4' 39. Qh3+" Ke5' 40. Qh2+' Qf4' 41. Qb8+' Ke6' 42. Qhxf4 [Qe2+] {KQQKQ} Qa3+' 43. Qb3+' Qxb3+' 44. Kxb3" {KQK} Kd5^3 45. Kb4' Kc6^2 46. Qd2 Kb7 47. Qd7+' Ka8^3 48. Ka5^3 Kb8° 49. Ka6^2 Ka8° 50. Qa7#^{5}.

In the 42-move 4Q phase proper here, White has only one winning move 16 times and a two-way choice of optimals 3 times. Black has one legal move twice, a two-way choice of optimals once and interposes a Queen 14 times.

White need not capture a Queen but could continue 42. Qe2+ Kd7 [Kf5, Kf7] 43. Qa7+ [Qb7+, Qxe7+] Kc6 [Qc7+] 44. Qaa6+' Kd5' 45. Qaa2+ [Qad3+, Qea2+] Kc6 [Kd6] 46. Qea6+' Kc7 [Kd7] 47. Q2a5+' Kd7' 48. Q5b5+' Kc7 [Kd8] 49. Qbb7+ [Qbc6+] Kd8° 50. Qc8#, a just-in-time 50-move 4Q mate.

A DTC line for position M3 branches off after 29. Qcc5+' with 29. ... Kg7' 30. Qa7+' Q8e7' 31. Qg1+' Kf6' 32. Qb6+' Q7e6' 33. Qh6+' Kf7' 34. Qf2+' Q6f5' 35. Qh7+' Kf6' 36. Qb6+' Kg5' 37. Qhh6+ [Qd8+] Kg4° 38. Qg1+" Kf3' 39. Qhh1+ [Qgh1+] Kf4' 40. Qhh2+' Kf3° 41. Qhf2# [Qgg3#], again a 4Q mate.

B2 Eight Mutual Zugzwangs

Both sides are optimising DTM; on balance, Black sacrifices a Queen earlier than it would with strategy C.

Z1, btm: 1. ... Qh1' 2. Qg7" Qb8+' 3. Kxb8" {KQQKQ} Qh2+' 4. Ka8 [Kc8, Qgg3, Qag3] Qg2+' 5. Qb7+" {not 4.Qxg2?=} Qxb7+' 6. Kxb7° Kc2 7. Qe3^5 {and mate on move 13}.

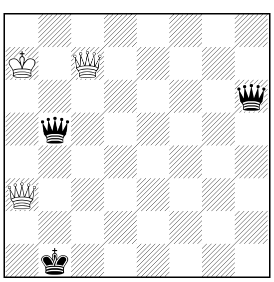
Z2, btm: 1. ... Qe3+' 2. Kxe3° {KQQKQ} Qg1+' 3. Kd2' Qg5+' 4. Qf4' Qg2+' 5. Ke3' Qh3+ [Qg1+] 6. Qf3 [Kf2] Qe6+ [Qxf3+, Qh6+] 7. Kf2' Qb6+' 8. Kf1' Qc5 [Qa6+, Qb4, Qb5+, Qe3, Qf6, Qg1+] 9. Qab3+ [Qfb3+, Qe2+, Qg2+] Ka1' 10. Qa8+ [Qfd1+, Qf6+] Qa3 [Qa5, Qa7] 11. Qaxa3#.

Z3, btm: 1. ... Qaa6' 2. Qhg1+" Qaf1 [Qff1] 3. Q1xf1+ [Q2xf1+] {KQQKQ} Qxf1+° 4. Qxf1+" Kb2' 5. Qd3^7 Ka1 [Ka2, Kc1] 6. Qd2^6 {and mate on move 12}.

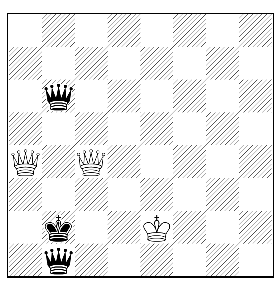
Z4, btm: 1. ... Qf7 [Qe7+] 2. Qd3+' Ke1' 3. Qc3+' Kd1' 4. Qa1+' Ke2' 5. Qg2+' Qff2' 6. Qe5+' Kd1 [Kd2] 7. Qgd5+' Qhd4' 8. Qdxd4+ [Qexd4+] {KQQKQ} Qxd4' 9. Qxd4+" Ke1' 10. Qf4^4 Ke2' 11. Kb3^5 Kd3 [Ke1] 12. Kb2 [Qe5] Ke2° 13. Kc2 [Kc3] Ke1° 14. Kd3' Kd1° 15. Qd2# [Qf1#].

Z5, btm: 1. ... Q1e2' 2. Qb3+' Ke1' 3. Qc3+' Kf1' 4. Qh1+' Qg1° 5. Qc1+" Qe1' 6. Qxe1' {KQQKQ} Kxe1° 7. Qxg1" Kd2 [Ke2] 8. Qg3^6 Ke2' 9. Kc2 [Kc1, Qf4] {and mate on move 13}.

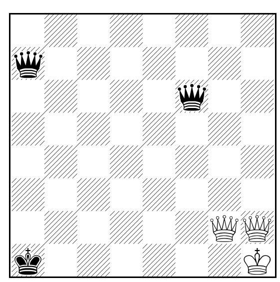
Z6, btm: 1. ... Qa2' 2. Qh6+" Kd1' 3. Qd6+' Kc1' 4. Qf4+' Kc2' 5. Qh2+' Qd2' 6. Qc5+ [Qc6+] Kb1 [Kd1] 7. Qcg1+" Kc2' 8. Qxd2+' {KQQKQ} Kxd2' 9. Qf2+ [Qg2+, Qh2+] {skewer} Kc3 [Kd3] 10. Qxa2" Kb4 [Kd3, Kd4] 11. Qc2' {and mate on move 16}.



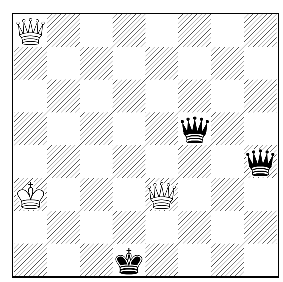
Zugzwang pos. **Z1**



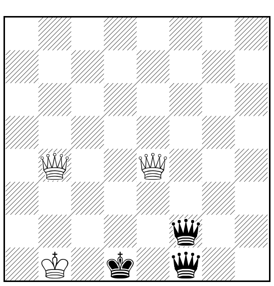
Zugzwang pos. **Z2**



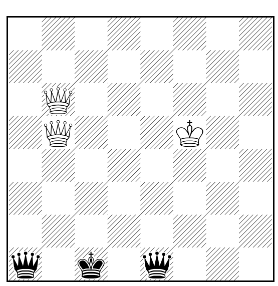
Zugzwang pos. **Z3**



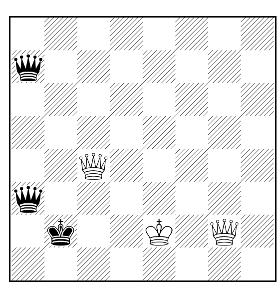
Zugzwang pos. **Z4**



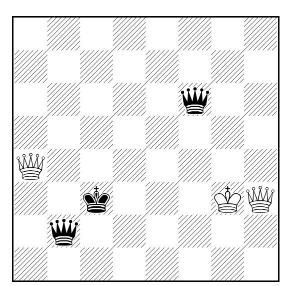
Zugzwang pos. **Z5**



Zugzwang pos. **Z6**



Zugzwang pos. **Z7**



Zugzwang pos. **Z8**

Z7, btm: 1. ... Qd7' 2. Qf2" {maintaining the Royal Battery} Qb3' 3. Kf1+' Ka3' 4. Qfc5+' Ka2' 5. Qa5+' Kb2' 6. Qe5+' Ka2' 7. Qee2+' Ka3' 8. Qc5+' Qb4' 9. Qc1+' Ka4' 10. Qa1+' Qa3' 11. Qc4+ [Qa6+] Ka5° 12. Qxa3+' {KQQKQ} Qa4' 13. Qcb4+ [Qaxa4+, Qac5+] Ka6° 14. Qaxa4#'.

Z8, btm: 1. ... Qe5+' 2. Kg4+" Kd2' 3. Qh6+" Kc3' 4. Qac6+' Kb3' 5. Qb7+" Kc2' 6. Qg2+ [Qbh7+, Qhh7+] Kb3' 7. Qb6+' Ka2' 8. Qa6+' Kb3' 9. Qf3+' Qbc3' 10. Qd1+" Qc2' 11. Qad3+" Kb4' 12. Q1xc2' {KQQKQ} Qg7+ [Qe6+] 13. Kf3' Qf6+ [Qf7+] 14. Qf5' Qc3+ [Qxf5+] 15. Qxc3+' {and mate on move 21}.

B3 Chess Game: Kasparov-World

The ! and ? comments are from the World Team's commentary (Krush and Regan, 1999). A KQPKQP≈ table not recognising Pawn-underpromotion (Karrer, 1999) indicates that, after moves 55. Qxb4, 58. ... Qe4 and 60. ... Kc1, Kasparov had mates in 82, 40 and 30 moves, finishing the game on moves 137, 98 and 90 respectively.

1. e4 c5 2. Nf3 d6 3. Bb5+ Bd7 4. Bxd7+ Qxd7 5. c4 Nc6 6. Nc3 Nf6 7. 0-0 g6 8. d4 cxd4 9. Nxd4 Bg7 10. Nde2 Qe6! 11. Nd5! Qxe4 12. Nc7+ Kd7 13. Nxa8 Qxc4 14. Nb6+ axb6 15. Nc3! Ra8 16. a4! Ne4! 17. Nxe4 Qxe4 18. Qb3 f5! 19. Bg5 Qb4! 20. Qf7 Be5 21. h3! Rxa4! 22. Rxa4 Qxa4 23. Qxh7 Bxb2 24. Qxg6 Qe4 25. Qf7 Bd4 26. Qb3 f4! 27. Qf7 Be5 28. h4 b5 29. h5 Qc4! 30. Qf5+ Qe6 31. Qxe6+ Kxe6 32. g3 fxg3 33. fxg3 b4! 34. Bf4!? Bd4+ 35. Kh1! b3 36. g4 Kd5! 37. g5 e6! 38. h6!? Ne7 39. Rd1 e5 40. Be3 Kc4 41. Bxd4 exd4 42. Kg2 b2 43. Kf3 Kc3 44. h7 Ng6 45. Ke4 Kc2 46. Rh1 d3 47. Kf5 b1=Q 48. Rxb1 Kxb1 49. Kxg6 d2 50. h8=Q d1=Q {KQPKQPP: 7Q/1p6/3p2K1/6P1/8/8/8/1k1q4+w, pos. GK1} 51. Qh7! b5? 52. Kf6+ Kb2? 53. Qh2+ Ka1 54. Qf4 b4?? 55. Qxb4 {KQPKQP: 8/8/3p1K2/6P1/1Q6/8/8/k2q4+b, pos. GK2} Qf3+ 56. Kg7" d5 57. Qd4+!" Kb1' 58. g6" {pos. GK3} Qe4? [Qf5'] 59. Qg1+' Kb2 60. Qf2+' Kc1 [Ka1'] 61. Kf6' d4' 62. g7' 1-0.

B4 Chess Analysis

B4.1 Künitz-Baez (1992, ECO A21) after 57. ... b1=Q, 6Q1/3K1Q2/k7/8/4q3/8/8/1q6+w. White asked the best question with 58. Qf6+ requiring the reply 58. ... Ka7". Black in fact played 58. ... Qb6?, leaving a 14-move win. White returned the favour with 59. Qf1+? instead of 59. Qa1+", leaving a draw again. The point was halved at this stage. M-optimal play from 58. ... Qb6 is:

59. Qa1+" Qa5' 60. Qc8+" Kb5' 61. Qb8+' Kc4 [Qb6] 62. Qc1+' Qc3' 63. Qc8+' Kb5 [Kd5] 64. Q1xc3' {KQQKQ} Qd5+^{4} 65. Kc7' Qc4+ [Qc5+] 66. Kb8' Qxc3' 67. Qxc3" {KQK} Kb6' 68. Kc8^{6} Kb5' 69. Kc7' and mate in three.

B4.2 Kasparov-World (1999, ECO B52). This is an equi-optimal line chosen (Tsaturjan, 1999) for its curiosity value: Black has just one escape route eight times and then forces a KQQK stalemate. The best choice of the many equi-optimal moves requires an opponent-modelling strategy to leave the greatest difficulties on the next move.

Pos. **GK1**: 51. Qh7 d5 52. Qxb7+ Ka1 53. Kh6 d4 54. g6 d3 55. g7 Qc1+ 56. Kh7 d2 57. g8Q Qc2+ 58. Kh8 d1Q {4Q: 6QK/1Q6/8/8/8/8/2q5/k2q4+w =} 59. Qga8+^{14} Qa2 [Qa4] 60. Qg7+^{19} Kb1" 61. Qe4+^{12} Qac2" 62. Qgb7+^{18} Ka2 [Ka1] 63. Qa6+^{17} Kb1" 64. Qb5+^{14} Ka1 [Kc1] 65. Qa5+^{11} Qa2" 66. Qee5+^{11} Kb1" 67. Qab5+^{11} Qab3" 68. Qe4+^{16} Kb2 [Ka1, Ka2, Kc1] 69. Qbe5+^{16} Ka2" 70. Qa8+^{17} Qa3 [Qa4] 71. Qh2+^{15} Kb1" 72. Qxa3^{10} {but now Black forces the draw in two} Qd4+ [Qd8+] 73. Kg8 [Kh7] Qg7+ [Qc4+, Qd5+, Qg4+] 74. Kxg7°= {KQQK}.

B5 Chess Art

Here are the two known studies (PCCC, 1999) featuring 4Q in the main line and one study featuring 4Q only in sublines. Nalimov confirms the 4Q positions as won and reproduces Elkies' optimal play. In the second study (Pospíšil, 1976), 8. ... Q1d3 and 9. Qc1+ are M-suboptimal and there is an attractive optimal line after 8. ... Q1d3. Elkies notes the similarity between this study and the game above.

B5.1 N.D. Elkies, Amer. Chess J. (1993), later reprinted (Rusinek, 1994, #546; Stiller, 1995, p. 107 and 1996, p. 177), 5Q2/5P1b/8/7K/8/1q4k1/1p4B1/8+w:

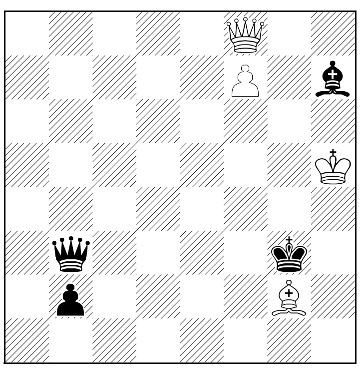
The author’s analysis: 1. Qg7+ (1. Qd6+? Kxg2 2. f8=Q Qh3+ 3. Kg5 Qe3+ forcing perpetual check or a Q-trade) 1. ... Kh2 2. f8=Q (2. Qe5+ Kxg2 3. f8=Q Qh3+ 4. Kg5 b1=Q with Kh1 and Be4 draws) 2. ... Qb5+ (not 2. ... b1=Q 3. Qf4+ Kg1 4. Be4+ and mate, or 2. ... Qd1+? 3. Bf3) 3. Kh6 Qb6+ 4. Bc6! (4. Kxh7? b1=Q+ 5. Kh8 Qb8! =) 4. ... Qxc6+ (4. ... Qe3+ 5. Qg5 Qxg5+ 6. Kxg5 b1=Q 7. Qf2+ mates) 5. Kxh7 b1=Q+ {5Q2/6QK/2q5/8/8/8/7k/1q6+w: 4Q, Bl. having checked first} 6. Kh8" Kh1 [Qg2] (6. ... Qg2 7. Qc7+ Kg1' 8. Qfc5+" Kh1' 9. Qh5+" wins) 7. Qfg8" {≡ pos. Z3} Qbc1' 8. Q8h7+" Q1h6 [Q6h6] 9. Qgxh6+[2] and wins

B5.2 J. Pospíšil (1976), study #1476 entered in tourney (1975), 2K5/4P2Q/8/1k2qp2/3p4/8/8/8+w, Wh. to win:

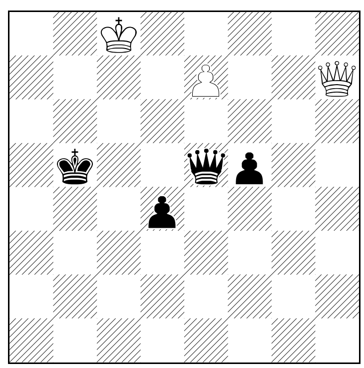
The author’s analysis: 1. Qf7! (1. Kd7? Qd5+ 2. Ke8 d3 3. Kf8 Qd6 4. Qxf5+ *(4. Kg8 Qb8+ 5. Kf7 Qc7 6. Kf6 Qc6+)* Kc4! 5. Kf7 Qc7 6. Qe4+ Kc3 7. Kg8 Qg3+ 8. Kf8 Qd6 9. Qe3 Kc2 10. Kg7 Qd7) 1. ... Qc5+ 2. Kd8 Qd6+ 3. Ke8 Kb4! (3. ... d3 4. Qb3+ Ka5 5. Kf7 Qd7 6. Qa3+ Kb6 7. Kf8 d2 8. Qb3+ Kc5 9. e8=Q d1=Q 10. Qc3+) 4. Qxf5 d3 (4. ... Kc3 5. Kf7 Qc7 6. Qf6 Qb7 *(6. ... Kc4 7. Kg7 Qg3+ 8. Kf8 Qa3 9. Qc6+; 6. ... Qc4+ 7.Qe6 Qc7* (7. ... Qf1+ 8. Kg7 Qg2+ 9. Qg6 Qb7 10. Qf7 Qg2+ 11. Kf8) *8. Qe1+ Kb2 9. Kg8 Qc4+ 10. Kh7)* 7. Kf8 Qb4 8. Qe5! Kc2 9. Qe4+ Kc3 10. Kg7) 5. Kf7 d2 6. e8=Q d1=Q {4Q: 4Q3/5K2/3q4/5Q2/1k6/8/8/3q4+w} 7. Qfb5+' Kc3' 8. Qe3+' Q1d3 9. Qc1+ Kd4' (9. ... Qc2 10. Qbb2+') 10. Qxd3+ Kxd3+' 11. Qd1+" and wins.

M-optimal branches are 8. ... Q6d3' 9. Qee5+' Qd4' 10. Qbc5+' Kb2 [Kb3] 11. Qcxd4+ [Qexc4+] Qxd4' 12. Qxd4+' with a KQK win, and after 8. ... Q1d3, 9. Qe1+' Kd4' 10. Qf2+' Qe3' 11. Qa4' Kd5' 12. Qf5+' Qde5' 13. Qfd7+' Qd6' 14. Qac6+' Ke5' 15. Qcxd6+ [Qdxd6+] {KQQKQ} Ke4° 16. Qg4+' Qf4+° 17. Qgxf4#'.

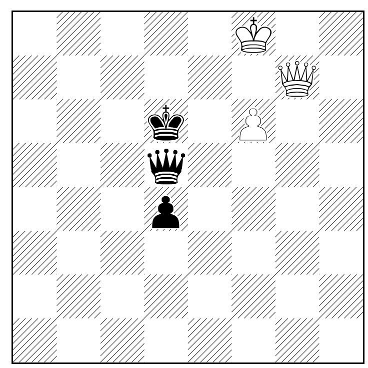
The 4Q EGT therefore cooks the study from move 7 where 7.Qeb5+ also wins even if it is suboptimal by 7 moves. Alternative winning moves in the main line are 8.Qc8+/Qh8+, 9. Qe1+' and 10.Qa1+/Qa4+'/Qg1+. The study is best repaired by ending the line at move six unless a subline can be promoted to be the Principal Variation.



B5.1: Elkies (1993). wtm and win



B5.2: Pospíšil (1976). wtm and win



B5.3: Pospíšil (1973). wtm and win

B5.3 J. Pospíšil (1973), study #1359, 2nd. hon. mention, 5K2/6Q1/3k1P2/3q4/3p4/8/8/8+w, Wh. to win:

1. Qg4! (1. Qe7+ Kc6 2. f7 d3 3. Ke8 d2 4. f8=Q Qh5+/i) 1. ... d3 (1. ... Kc7 2. f7 d3 3. Kg7 Qe5+ 4. Kh7 Qd6 5. Qg7 Qh2+ 6. Kg8 Qa2 7. Qg3+ Kc6 8. Qxd3/ii Qg2+ 9. Kf8) 2. f7 (2. Kg7? Qe5 3. Kg6 Qe2! 4. Qg3+! Kd7! 5. f7 Qe4+ 6. Kg7 Qd4+ 7. Kg8 Qd5) 2. ... d2 (2. ... Kc7 3. Kg7 Qe5+ 4. Kh7 Qd6 5. Qg7 Qh2+ 6. Kg8 Qa2 7. Qg3+ Kc6 8. Qxd3/ii) 3. Kg7 Qe5+ (3. ... d1=Q 4. f8=Q+/iii Kc6' 5. Qfc8+ Kb5' 6. Qb8+" Ka6' 7. Qg6+/iv Ka5 8. Qgb6+' Ka4° 9. Qb4#') 4. Kg6 Qe6+ (4. ... Qe7 5. Qb4+ Ke6 6. f8=N+! Kd5 7. Qxe7 d1=Q 8. Qd7+) 5. Qxe6+ Kxe6' 6. f8=Q" d1=Q 7. Qe8+" Kd6° 8. Qd8+". Some notes by this paper’s third author:

- i 5. Qef7 (5.Kd8 d1=Q+ {4Q and Bl. mates in 5} 6. Kc8' Qdg4+ [Qhg4+] 7. Kd8 [Qf5] Qd5+' 8. Qd6+' Qxd6+" 9. Qxd6+' Kxd6" 10. Ke8° Qg8#') Qxf7+ 6. Qxf7 d1=Q {KQKQ} =.
- ii White has a KQPKQ mate in 21. After 8. ... Qg2, 9. Kh7' is optimal but 9. Kf8 adds 11 moves to the win.
- iii btm 4Q position but White mates in 10.
- iv obvious but not optimal; Black in fact avoids the queen sacrifice but hastens the mate. M-optimal was 7. Qgc8+' Ka5' 8. Qa7+ [Qc3+] Kb5' 9. Qaa6+ [Qcc7+] Kb4° 10. Qb6+' Qb5' 11. Qcc5+' Kb3' 12. Qbxb5+ [Qcxb5+] Ka2° 13. Ka7+' Qa4° 14. Qaxa4#'.